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Applied Mathematics Letters 19 (2006) 901–907

**Applied
Mathematics
Letters**www.elsevier.com/locate/aml

Exponential stability of uncertain dynamic systems including state delay

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Received 30 September 2005; accepted 24 October 2005

Abstract

In this letter, the exponential stability of uncertain time-delay systems is investigated. Based on the Lyapunov method, a novel stability criterion has been derived in terms of matrix inequalities which can be easily solved using efficient convex optimization algorithms. Two numerical examples are included to show the effectiveness of the proposed method.

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Keywords: Time-delay system; Exponential stability; Lyapunov method; Convex optimization

1. Introduction

Time delay often occurs in many practical systems such as population models, chemical processes, biological and economic systems and frequently is a source of instability [1]. In view of this, the stability issue of time-delay systems is a topic of great practical importance which has attracted a lot of interest over the decades. For more characteristics of the system, see the Refs. [1,2]. Also, system uncertainties arise from many sources such as unavoidable approximation, data errors and ageing of systems. So, the problem of robust stability analysis for uncertain time-delay systems has been investigated by many researchers [3–9].

In the literature [7–9], the results derived are delay-dependent stability criteria which are less conservative than delay-independent ones when the size of the time delay is small. However, the criteria only guarantee the asymptotic stability of several classes of time-delay systems, instead of exponential stability. Recently, Liu [10] investigated the delay-dependent exponential stability of linear time-delay systems. The uncertainties are not considered in the work [10].

In this letter, we propose a new robust exponential stability criterion which is delay dependent. To reduce the conservatism of the stability criterion, a new Lyapunov function which employs free weighting matrices is introduced. Utilizing a parameterized neutral model transformation which allows free variables in a certain operator, a new delay-dependent stability criterion has been proposed. This criterion is derived in terms of matrix inequalities which can be efficiently solved by using various convex optimization algorithms [11].

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Notation. \mathcal{R}^n is the n -dimensional Euclidean space. $\mathcal{R}^{m \times n}$ denotes the set of $m \times n$ real matrices. \star denotes the symmetric part. $X > 0$ ($X \geq 0$) means that X is a real symmetric positive definite matrix (positive semi-definite). I denotes the identity matrix with appropriate dimensions. $\text{diag}\{\cdot \cdot \cdot\}$ denotes the block diagonal matrix. $\mathcal{C}_{n,h} = \mathcal{C}([-h, 0], \mathcal{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h, 0]$ into \mathcal{R}^n , with the topology of uniform convergence.

2. Main results

Consider the systems described by the following state equation:

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t-h), \\ x(s) &= \phi(s), \quad s \in [-h, 0],\end{aligned}\tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $A \in \mathcal{R}^{n \times n}$, $A_1 \in \mathcal{R}^{n \times n}$ are known real parameter matrices, $h > 0$ is a constant delay, $\phi(s) \in \mathcal{C}_{n,h}$ is a given continuous vector valued initial function. The time-varying parameter uncertainties ΔA and ΔA_1 are assumed to be in the form [4–6,9] of

$$\Delta A = DF(t)E, \quad \Delta A_1 = D_1F_1(t)E_1,\tag{2}$$

where D, D_1, E and E_1 are known real constant matrices of appropriate dimensions, $F(t) \in \mathcal{R}^{k \times l}$ and $F_1(t) \in \mathcal{R}^{k_1 \times l_1}$ are unknown matrices, which satisfy

$$F^T(t)F(t) \leq I, \quad F_1^T(t)F_1(t) \leq I.\tag{3}$$

Let us consider the following transformation:

$$z(t) = e^{\alpha t}x(t),\tag{4}$$

where the positive scalar α is the exponential stability degree (delay decay rate), for transforming system (1) into

$$\dot{z}(t) = (A + \Delta A + \alpha I)z(t) + e^{\alpha h}(A_1 + \Delta A_1)z(t-h).\tag{5}$$

The asymptotic stability of system (5), once it is established, implies that system (1) is exponentially stable with decay rate α .

Now, define an operator $\mathcal{D}(z_t) : \mathcal{C}_{n,h} \rightarrow \mathcal{R}^n$ as

$$\mathcal{D}(z_t) = z(t) + \int_{t-h}^t Ge^{\alpha h}z(s)ds,\tag{6}$$

where $z_t = z(t+s)$, $s \in [-h, 0]$ and $G \in \mathcal{R}^{n \times n}$ is a constant matrix which will be chosen.

With the above operator, the transformed system is

$$\begin{aligned}\dot{\mathcal{D}}(z_t) &= \dot{z}(t) + Ge^{\alpha h}z(t) - Ge^{\alpha h}z(t-h) \\ &= (A + \Delta A + \alpha I + Ge^{\alpha h})z(t) + e^{\alpha h}(A_1 + \Delta A_1 - G)z(t-h).\end{aligned}\tag{7}$$

Here, the following well-known facts and lemmas are needed for deriving the main results.

Fact 1. For given matrices D, E, F with $F^T F \leq I$ and scalar $\epsilon > 0$, the following inequality:

$$DFE + E^T F^T D^T \leq \epsilon DD^T + \epsilon^{-1} E^T E$$

is satisfied.

Fact 2 (Schur Complement). Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, we have $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 1 ([12]). For any constant matrix $M \in \mathcal{R}^{n \times n}$, $M = M^T > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \rightarrow \mathcal{R}^n$ such that the integrations concerned are well defined, we have

$$\left(\int_0^\gamma \omega(s) ds \right)^T M \left(\int_0^\gamma \omega(s) ds \right) \leq \gamma \int_0^\gamma \omega^T(s) M \omega(s) ds. \quad (8)$$

Lemma 2 ([14]). Consider an operator $\mathcal{D}(\cdot) : \mathcal{C}_{n,h} \rightarrow \mathcal{R}^n$ with $\mathcal{D}(x_t) = x(t) + \hat{B} \int_{t-h}^t x(s) ds$, where $x(t) \in \mathcal{R}^n$ and $\hat{B} \in \mathcal{R}^{n \times n}$. For a given scalar δ , where $0 < \delta < 1$, if a positive definite symmetric matrix M exists satisfying the inequality

$$\begin{bmatrix} -\delta M & h\hat{B}^T M \\ hM\hat{B} & -M \end{bmatrix} < 0, \quad (9)$$

then the operator $\mathcal{D}(x_t)$ is stable.

Then, we have the following theorem for exponential stability of system (1).

Theorem 1. For given h , α , and $\beta > 1$, system (1) is exponentially stable with decay rate α if there exist positive definite matrices P , T , F_{11} , F_{22} , F_{33} , positive scalars ε_i ($i = 1, \dots, 4$), and any matrices Y , F_{12} , F_{13} , F_{23} which satisfy the following inequalities:

$$\begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & 0 & 0 & PD & e^{\alpha h} PD_1 & \beta h e^{\alpha h} Y^T \\ \star & -h^{-1}(\beta - 1)P & \Pi_4 & PD & e^{\alpha h} PD_1 & 0 & 0 & 0 \\ \star & \star & \Pi_5 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\varepsilon_3 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\varepsilon_4 I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\varepsilon_1 I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\varepsilon_2 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\beta h P \end{bmatrix} < 0, \quad (10)$$

$$\begin{bmatrix} -P & h e^{\alpha h} Y^T \\ \star & -P \end{bmatrix} < 0, \quad (11)$$

$$-P + F_{22} < 0, \quad (12)$$

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ \star & F_{22} & F_{23} \\ \star & \star & F_{33} \end{bmatrix} > 0, \quad (13)$$

where

$$\Pi_1 = PA + A^T P + 2\alpha P + e^{\alpha h}(Y + Y^T) + T + hF_{11} + (\varepsilon_1 + \varepsilon_3)E^T E,$$

$$\Pi_2 = A^T P + \alpha P + e^{\alpha h} Y^T + F_{12},$$

$$\Pi_3 = e^{\alpha h}(PA_1 - Y) + hF_{13},$$

$$\Pi_4 = e^{\alpha h}(PA_1 - Y) + F_{23},$$

$$\Pi_5 = -T + hF_{33} + (\varepsilon_2 + \varepsilon_4)E_1^T E_1.$$

Proof. Consider the legitimate Lyapunov function candidate [1]

$$V = V_1 + V_2 + V_3 + V_4 \quad (14)$$

where

$$V_1 = \mathcal{D}^T(z_t) P \mathcal{D}(z_t), \quad (15)$$

$$V_2 = \beta e^{2\alpha h} \int_{t-h}^t \int_s^t z^T(u) G^T P G z(u) du ds, \quad (16)$$

$$V_3 = \int_{t-h}^t z^T(s) T z(s) ds, \quad (17)$$

$$V_4 = \int_0^t \int_{s-h}^s \begin{bmatrix} z(s) \\ e^{\alpha h} G z(u) \\ z(s-h) \end{bmatrix}^T \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ \star & F_{22} & F_{23} \\ \star & \star & F_{33} \end{bmatrix} \begin{bmatrix} z(s) \\ e^{\alpha h} G z(u) \\ z(s-h) \end{bmatrix} du ds. \quad (18)$$

Taking the time derivative of V gives that

$$\begin{aligned} \dot{V}_1 &= 2\mathcal{D}^T(z_t) P \dot{\mathcal{D}}(z_t) \\ &= 2 \left\{ z(t) + \int_{t-h}^t G e^{\alpha h} z(s) ds \right\}^T P \left\{ (\bar{A} + \Delta A) z(t) + e^{\alpha h} (A_1 + \Delta A_1 - G) z(t-h) \right\} \\ &= z^T(t) [P\bar{A} + \bar{A}^T P] z(t) + 2z^T(t) P e^{\alpha h} (A_1 - G) z(t-h) \\ &\quad + 2 \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T P \bar{A} z(t) + 2 \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T P e^{\alpha h} (A_1 - G) z(t-h) \\ &\quad + 2z^T(t) P \Delta A z(t) + 2z^T(t) P e^{\alpha h} \Delta A_1 z(t-h) \\ &\quad + 2 \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T P \Delta A z(t) + 2 \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T P e^{\alpha h} \Delta A_1 z(t-h), \end{aligned} \quad (19)$$

$$\begin{aligned} \dot{V}_2 &= \beta h z^T(t) e^{2\alpha h} G^T P G z(t) - \beta \int_{t-h}^t z^T(s) e^{2\alpha h} G^T P G z(s) ds \\ &\leq \beta h z^T(t) e^{2\alpha h} G^T P G z(t) - \int_{t-h}^t z^T(s) e^{2\alpha h} G^T P G z(s) ds \\ &\quad - h^{-1}(\beta - 1) \left(\int_{t-h}^t G e^{\alpha h} z(s) ds \right)^T P \left(\int_{t-h}^t G e^{\alpha h} z(s) ds \right), \end{aligned} \quad (20)$$

$$\dot{V}_3 = z^T(t) T z(t) - z^T(t-h) T z(t-h), \quad (21)$$

$$\begin{aligned} \dot{V}_4 &= h z^T(t) F_{11} z(t) + 2z^T(t) F_{12} \int_{t-h}^t G e^{\alpha h} z(s) ds + 2h z^T(t) F_{13} z(t-h) \\ &\quad + \int_{t-h}^t z^T(s) e^{2\alpha h} G^T F_{22} G z(s) ds + 2 \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T F_{23} z(t-h) \\ &\quad + h z^T(t-h) F_{33} z(t-h), \end{aligned} \quad (22)$$

where $\bar{A} = A + \alpha I + G e^{\alpha h}$, and [Lemma 1](#) is utilized in (20).

Using [Fact 1](#), several terms of the right-hand side of (19) are bounded:

$$2z^T(t) P D F(t) E z(t) \leq \varepsilon_1^{-1} z^T(t) P D D^T P z(t) + \varepsilon_1 z^T(t) E^T E z(t), \quad (23)$$

$$2e^{\alpha h} z^T(t) P D_1 F_1(t) E_1 z(t-h) \leq \varepsilon_2^{-1} e^{2\alpha h} z^T(t) P D_1 D_1^T P z(t) + \varepsilon_2 z^T(t-h) E_1^T E_1 z(t-h), \quad (24)$$

$$\begin{aligned} &2 \left(\int_{t-h}^t e^{\alpha h} G x(s) ds \right)^T P D F(t) E z(t) \\ &\leq \varepsilon_3^{-1} \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T P D D^T P \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right) + \varepsilon_3 z^T(t) E^T E z(t), \end{aligned} \quad (25)$$

$$\begin{aligned} &2 \left(\int_{t-h}^t e^{\alpha h} z x(s) ds \right)^T P e^{\alpha h} D_1 F_1(t) E_1 z(t-h) \\ &\leq \varepsilon_4^{-1} \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right)^T e^{2\alpha h} P D_1 D_1^T P \left(\int_{t-h}^t e^{\alpha h} G z(s) ds \right) \\ &\quad + \varepsilon_4 z^T(t-h) E_1^T E_1 z(t-h). \end{aligned} \quad (26)$$

From (19)–(26), the time derivative of V has a new upper bound as follows:

$$\begin{aligned} \dot{V} \leq & \begin{bmatrix} z(t) \\ \int_{t-h}^t e^{\alpha h} G z(s) ds \\ z(t-h) \end{bmatrix}^T \Omega \begin{bmatrix} z(t) \\ \int_{t-h}^t e^{\alpha h} G z(s) ds \\ z(t-h) \end{bmatrix} \\ & + \int_{t-h}^t e^{2\alpha h} z^T(s) G^T (-P + F_{22}) G z(s) ds, \end{aligned} \quad (27)$$

where

$$\Omega = \begin{bmatrix} \Omega_{1,1} & \bar{A}^T P + F_{12} & P e^{\alpha h} (A_1 - G) + h F_{13} \\ \star & \Omega_{2,2} & P e^{\alpha h} (A_1 - G) + F_{23} \\ \star & \star & -T + h F_{33} + (\varepsilon_2 + \varepsilon_4) E_1^T E_1 \end{bmatrix} \quad (28)$$

where $\Omega_{1,1} = P\bar{A} + \bar{A}^T P + T + \beta h e^{2\alpha h} G^T P G + h F_{11} + \varepsilon_1^{-1} P D D^T P + \varepsilon_1 E^T E + \varepsilon_2^{-1} e^{2\alpha h} P D_1 D_1^T P + \varepsilon_3 E^T E$, and $\Omega_{2,2} = -h^{-1}(\beta - 1)P + \varepsilon_3^{-1} P D D^T P + \varepsilon_4^{-1} e^{2\alpha h} P D_1 D_1^T P$.

Hence, if $\Omega < 0$ and $-P + F_{22} < 0$, then a positive scalar λ exists which satisfies

$$\dot{V} < -\lambda \|z(t)\|^2. \quad (29)$$

Let $Y = PG$. By using Fact 2 (the Schur Complement), the resulting inequality is equivalent to (10). If inequality (11) holds, then we can prove that a positive scalar δ which is less than one exists such that

$$\begin{bmatrix} -\delta P & h e^{\alpha h} G^T P \\ \star & -P \end{bmatrix} < 0 \quad (30)$$

according to matrix theory. Therefore, from Lemma 2, if the inequality (11) holds, then operator $\mathcal{D}(z_t)$ is stable. The inequality (13) means that V_4 is non-negative. According to the Theorem 9.8.1 in [1], we conclude that if matrix inequalities (10)–(13) hold, then system (5) is asymptotically stable. This guarantees the exponential stability with decay rate α of system (1), which completes our proof. \square

Remark 1. By solving iteratively the LMIs of Theorem 1 with respect to α , one can obtain the maximum allowable decay rate α for guaranteeing exponential stability of system (1).

Remark 2. In this letter, we use the operator $\mathcal{D}(z_t) = z(t) + \int_{t-h}^t e^{\alpha h} G z(s) ds$ to transform the original system. Note that if G is A_1 , then the transformation is the neutral model transformation [1]. Since the operator $\mathcal{D}(z_t)$ has free weighting matrix, it is less conservative than the results obtained by using the neutral model transformation.

Remark 3. The solutions of Theorem 1 can be obtained by solving the generalized eigenvalue problem in $X, W, F_{11}, F_{33}, Y, F_{12}, F_{13}, F_{23}, \varepsilon_i$ ($i = 1, \dots, 4$), which is a quasiconvex optimization problem. Note that a locally optimal point of a quasiconvex optimization problem with strictly quasiconvex objective is globally optimal [11]. In this letter, we utilize Matlab's LMI Control Toolbox [13] which implements interior-point algorithms. These algorithms are significantly faster than classical convex optimization algorithms [11].

Example 1. Consider the following uncertain time-delay systems:

$$\dot{x}(t) = (A + DF(t)E)x(t) + (A_1 + D_1 F_1(t)E_1)x(t-1), \quad (31)$$

where

$$\begin{aligned} A &= \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}, & A_1 &= \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0.5 \end{bmatrix}, & D &= D_1 = I, \\ E &= E_1 = 0.2I, & F^T(t)F(t) &\leq I, & F_1^T(t)F_1(t) &\leq I. \end{aligned} \quad (32)$$

Then, we are going to find the maximum allowable decay rate on α for system (31). Applying Theorem 1 to the above system gives that the system is exponentially stable with $\alpha = 0.1794$. In the case of $\alpha = 0.1794$, the solutions of

Table 1

Stability bound of h for various stability degrees $0 \leq \alpha \leq 0.5$

	α	0	0.1	0.2	0.3	0.4	0.5
Ours	h	∞	11.498	5.525	3.584	2.649	2.111
Liu [10]	h	0.9643	0.7580	0.5411	0.4074	0.2809	0.1858

Table 2

Stability bound of h for various stability degrees $0.6 \leq \alpha \leq 0.9$

	α	0.6	0.7	0.8	0.9
Ours	h	1.765	1.524	1.345	1.191
Liu [10]	h	0.1243	0.0769	0.0482	0.0263

LMIs in Theorem 1 for a given scalar $\beta = 1224.2$ are as follows:

$$\begin{aligned}
P &= 10^4 \times \begin{bmatrix} 2.6676 & 2.2827 \\ 2.2827 & 3.7000 \end{bmatrix}, & T &= 10^4 \times \begin{bmatrix} 5.5441 & 2.0605 \\ 2.0605 & 2.6936 \end{bmatrix}, \\
Y &= \begin{bmatrix} 6.7334 & -7.9298 \\ 7.9388 & -9.3494 \end{bmatrix}, & F_{11} &= 10^4 \times \begin{bmatrix} 0.7507 & 0.8671 \\ 0.8671 & 1.4023 \end{bmatrix}, \\
F_{12} &= \begin{bmatrix} 64.5925 & 76.3454 \\ 40.4736 & 66.0404 \end{bmatrix}, & F_{13} &= 10^4 \times \begin{bmatrix} -1.0702 & -0.4908 \\ -1.3101 & -0.8806 \end{bmatrix}, \\
F_{22} &= 10^4 \times \begin{bmatrix} 2.4086 & 2.4794 \\ 2.4794 & 3.5160 \end{bmatrix}, & F_{23} &= \begin{bmatrix} -43.5884 & -26.9942 \\ -59.7100 & -38.0528 \end{bmatrix}, \\
F_{33} &= 10^4 \times \begin{bmatrix} 1.5402 & 0.7566 \\ 0.7566 & 0.5675 \end{bmatrix}, & \varepsilon_1 &= 10^5 \times 2.7122, & \varepsilon_2 &= 10^5 \times 3.2451, \\
\varepsilon_3 &= 198.2649, & \varepsilon_4 &= 237.2205.
\end{aligned}$$

Example 2. Consider the time-delay systems [10]

$$\dot{x}(t) = Ax(t) + A_1x(t-h), \quad (33)$$

where

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix}.$$

By applying Theorem 1 to system (33), the allowable bound on h with respect to α is obtained as given in Tables 1 and 2. Also, we compared our results with recent results in [10]. From Tables 1 and 2, one can see that our result gives more stability bounds on h than that of Liu [10].

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